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Delay-range-dependent control of nonlinear time-delay systems under input saturation

Muhammad Rehan[1,*,†], Naeem Iqbal[1] and Keum-Shik Hong[2]

[1]*Department of Electrical Engineering, Pakistan Institute of Engineering and Applied Sciences (PIEAS), Islamabad, Pakistan*
[2]*Department of Cogno-Mechatronics Engineering and School of Mechanical Engineering, Pusan National University; 2 Busandaehak-ro, Geumjeong-gu, Busan 609-735, Korea*

SUMMARY

This paper describes a delay-range-dependent local state feedback controller synthesis approach providing estimation of the region of stability for nonlinear time-delay systems under input saturation. By employing a Lyapunov–Krasovskii functional, properties of nonlinear functions, local sector condition and Jensen's inequality, a sufficient condition is derived for stabilization of nonlinear systems with interval delays varying within a range. Novel solutions to the delay-range-dependent and delay-dependent stabilization problems for linear and nonlinear time-delay systems, respectively, subject to input saturation are derived as specific scenarios of the proposed control strategy. Also, a delay-rate-independent condition for control of nonlinear systems in the presence of input saturation with unknown delay-derivative bound information is established. And further, a robust state feedback controller synthesis scheme ensuring L_2 gain reduction from disturbance to output is devised to address the problem of the stabilization of input-constrained nonlinear time-delay systems with varying interval lags. The proposed design conditions can be solved using linear matrix inequality tools in connection with conventional cone complementary linearization algorithms. Simulation results for an unstable nonlinear time-delay network and a large-scale chemical reactor under input saturation and varying interval time-delays are analyzed to demonstrate the effectiveness of the proposed methodology.

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KEY WORDS: state feedback control; input saturation; time-delay systems; delay-range-dependency; region of stability

1. INTRODUCTION

Nowadays, feedback control of time-delay systems is receiving substantial research attention owing to its countless potential and realized applications to network control systems, chemical processes, industrial plants, economics, neural networks, electronic circuits, secure communication, biological structures, population models, transmission lines, and electric power grids [1–5]. In a recent work [6], delay-dependent output feedback control achieving exponential stability for an electrical motor under missing data and time-varying network-induced lags was presented. Network control schemes that allow time-delays and data packet losses as well as data packet disorder and facilitate transmission control protocols were presented in [7, 8]. Control of stochastic time-delay systems under disturbances and noise has been carried out in [9, 10], and further, feedback control of linear as well as nonlinear systems under input or output time-delays for the desired closed-loop performance against actuator and measurement delays has been addressed in [11, 12].

*Correspondence to: Muhammad Rehan, Department of Electrical Engineering, Pakistan Institute of Engineering and Applied Sciences (PIEAS), Islamabad, Pakistan.
†E-mail: rehanqau@gmail.com

Control of linear and nonlinear time-delay systems by considering input nonlinearities, such as saturation constraint due to the bounded input limitation of actuators, has been remained a topic of research during the several past years. To address this problem, control of uncertain time-delay systems based on Razumikhin approach by taking constant unknown delays into account was studied in [13]. A dynamic anti-windup compensator design approach, by assuming a known nominal feedback controller, by ignoring saturating actuators, was provided in the work [14]. In [15], an adaptive control strategy under input nonlinearity was devised to ensure a reference tracking by enforcing the exponential convergence of the tracking error within a bounded region by employing a Lyapunov functional. A convex-constraints-based approach ensuring a large region of stability and the fast convergence of states in linear time-delay systems under actuator saturation was formulated in [16] by means of Lyapunov analysis and static anti-windup gains. Recently, some works such as [17, 18] studied adaptive neural network-based control schemes for interconnected large-scale systems by application of Lyapunov–Krasovskii functions, minimal-learning-parameters algorithms, and dynamic surface control theory. Further, an adaptive output feedback control methodology was explored in [19] by using neural networks for stochastic nonlinear time-delay systems with unknown control directions. However, many challenging issues have not been explored for the control of nonlinear time-delay systems under time-varying delays and in the presence of saturating actuators.

Delay-independent and delay-dependent feedback controller synthesis approaches for various applications have been formulated in most of the existing literature. Recently, new types of control methodologies for time-delay systems, known as delay-range-dependent schemes, which consider the delay-interval from a non-zero lower bound to a finite upper bound, have been introduced. For the first time, a delay-range-dependent control scheme for linear dynamical systems that provides sufficient conditions for computing the state feedback controller gain has been investigated in [20]. For uncertain linear systems, meanwhile, a delay-range-dependent stabilization approach based on a free weighting matrix and an augmented Lyapunov functional was developed in [21]. For uncertain linear time-delay systems containing delays in the state vector, a robust H_∞ state feedback controller design paradigm was investigated in [22]. Exponential stabilization results for linear systems subject to parametric uncertainties under non-differentiable interval delays were also available in the literature [23]. For stabilization of Takagi-Sugeno (TS) fuzzy systems, sufficient conditions that can be solved based on two coupling integral inequalities [24] and by using cone complementary linearization algorithms have been derived. Stabilization of linear time-delay systems with interval delays in the state, output, and input has been achieved using static and dynamic feedback control strategies [25, 26].

It is worth mentioning that most of the aforementioned delay-range-dependent control approaches are applicable to linear time-delay systems; however, stabilization of nonlinear time-delay systems with delays varying in an interval remains as an unresolved research problem. Indeed, in practical systems, input saturation is unavoidable because all actuators are operated through bounded input signals. Input saturation needs to be carefully considered when determining a control action; otherwise, it can stimulate the windup phenomenon and lead to an undesirable closed-loop system response, giving rise to lags, undershoots, overshoots, and instability. Improper treatment of actuator saturation has, because of the so-called windup aftermaths, caused airplane accidents and power plant failures (see, for instance, [27] and references therein). Consequently, obtainment of delay-range-dependent control strategies for nonlinear systems subject to input saturation constraints is a non-trivial research problem owing specifically to involvement of the three-fold difficulties of varying interval time-lags, dynamical nonlinearities, and actuator saturation limitations.

In this paper, a delay-range-dependent state feedback control scheme for nonlinear time-delay systems in the presence of input saturation is discussed. Sufficient conditions for controller synthesis are derived by exploiting the delay-interval, delay-derivative bound, and limits on nonlinear functions and by employing the Lyapunov–Krasovskii function, the local sector condition, and Jensen's inequality. The proposed controller design methodology ensures the local asymptotic convergence of the states within a guaranteed region of stability, provided that the initial conditions and their derivatives are selected from a bounded ellipsoidal region. To the best of our knowledge, a delay-range-dependent control approach to nonlinear time-delay systems under input saturation herein is provided for the first time. Moreover, novel delay-dependent and delay-intervaldependent results for nonlinear and linear systems, respectively, are inferred as specific cases of the proposed delay-range-dependent control treatment. Furthermore, a delay-rate-independent control scheme that supposes an interval time-varying delay is developed for control of nonlinear systems with inadequate delay-derivative information. Additionally, the robustness of the proposed control methodology to perturbations, which renders the L_2 gain reduction between the disturbance and the output of the nonlinear time-delay system, is offered. The resultant design conditions can be solved by employing cone complementary linearization algorithms and linear matrix inequality (LMI) techniques. Simulation results for the proposed delay-range-dependent control formulation are demonstrated herein for an unstable input-saturated nonlinear time-delay system and a large-scale chemical reactor under saturating actuator.

This paper is organized as follows: Section 2 describes the nonlinear time-delay system considered. Section 3 details the proposed state feedback control strategies for nonlinear systems under actuator saturation. Section 4 presents the simulation results. Section 5 concludes the paper.

Standard notation is employed throughout the paper. Euclidean and L_2 norms are symbolized by $\|x\|$ and $\|x\|_2$, respectively. $A_{(i)}$ denotes the ith row of a matrix A. $diag(x_1, x_2, \ldots, x_m)$ represents a diagonal matrix containing $x_1, x_2, \ldots, x_m$ at the respective diagonal entries. Symmetric positive-definite and symmetric positive-semi-definite matrices are denoted by the matrix inequalities $X > 0$ and $X \geqslant 0$, respectively. The saturation function for an input vector $u \in R^m$ is given by $sat(u) = \text{sign}(u_{(i)}) \min\left(\bar{u}_{(i)}, |u_{(i)}|\right)$, where $\bar{u}_{(i)} > 0$ refers to the ith bound on the saturation nonlinearity.

2. SYSTEM DESCRIPTION

Consider a nonlinear time-delay system given by

$$\begin{aligned} \frac{dx}{dt} &= Ax + A_d x(t-\tau) + f(t,x) + g(t, x(t-\tau)) + Bsat(u) + d, \\ y(t) &= Cx, \\ x(t) &= \phi(t), \quad t \in \left[-\tau_2 \quad 0\right], \end{aligned} \tag{1}$$

where $x \in R^n, u \in R^m, y \in R^p$, and $d \in R^n$ represent the state, input, output and disturbance vectors, respectively. The saturated control signal is denoted by $sat(u) \in R^m$. Delay in the state is symbolized by τ. The linear and the nonlinear dynamics are represented by matrices $A \in R^{n\times n}, A_d \in R^{n\times n}, B \in R^{n\times m}$, and $C \in R^{p\times n}$ and vector-functions $f(t,x) \in R^n$ and $g(t, x(t-\tau)) \in R^n$ (satisfying the condition $f(t,0) = g(t,0) = 0$), respectively. It is presumed that the time-delay satisfies the interval

$$0 \leqslant \tau_1 \leqslant \tau(t) \leqslant \tau_2, \tag{2}$$

and that the variation in the derivative of the time-delay is bounded by

$$\dot{\tau}(t) \leqslant \mu. \tag{3}$$

Assumption 1

The nonlinear dynamics in (1) satisfy

$$\|f(t,x) - f(t,\bar{x})\| \leqslant \left\|\Lambda_f (x - \bar{x})\right\|, \ \ \forall x, \bar{x} \in R^n, \tag{4}$$

$$\|g(t,x) - g(t,\bar{x})\| \leqslant \left\|\Lambda_g (x - \bar{x})\right\|, \ \ \forall x, \bar{x} \in R^n, \tag{5}$$

where Λ_f and Λ_g are constant matrices of an appropriate dimension.

It has been demonstrated [27–30] that the local sector condition

$$D_z^T(u) W \left[w - D_z(u)\right] \geqslant 0 \tag{6}$$

holds for a diagonal positive-definite matrix W, if an auxiliary defined region

$$S(\bar{u}) = \{w \in R^m, \ -\bar{u} \leqslant u - w \leqslant \bar{u}\} \tag{7}$$

is satisfied, where $\bar{u} \in R^m$ denotes the saturation bound, $w \in R^m$ refers to an auxiliary defined vector, and $D_z(u) = u - sat(u)$ represents the dead-zone vector-function.

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Lemma ([31, 32]*)*
Consider a positive-definite matrix $Z = Z^T$ and a positive scalar η. The integral inequality

$$\int_0^\eta \Gamma^T(\theta) Z \Gamma(\theta) d\theta \geqslant \eta^{-1} \left(\int_0^\eta \Gamma(\theta) d\theta \right)^T Z \left(\int_0^\eta \Gamma(\theta) d\theta \right)$$

holds, given that the integrals are well-defined, where $\Gamma(t)$ is a vector-function of appropriate dimension.

3. CONTROLLER SYNTHESIS UNDER INPUT SATURATION

We propose the following controller for stabilization of (1)

$$u = Fx(t). \tag{8}$$

The open-loop system (1) is rewritten as

$$\begin{aligned} \frac{dx}{dt} &= Ax + A_d x(t-\tau) + f(t,x) + g(t, x(t-\tau)) - BDz(u) + Bu + d, \\ y &= Cx, \end{aligned} \tag{9}$$

by application of $D_z(u) = u - sat(u)$. Incorporating (8) into (9) yields the closed-loop system

$$\begin{aligned} \frac{dx}{dt} &= (A + BF)x + A_d x(t-\tau) + f(t,x) + g(t, x(t-\tau)) - BDz(u) + d, \\ y &= Cx. \end{aligned} \tag{10}$$

Using (8) and selecting $w = Hx$, the region (7) and the local sector condition (6) are written as

$$S(\bar{u}) = \{ w \in R^m, -\bar{u} \leqslant (F - H)x \leqslant \bar{u} \}, \tag{11}$$

$$D_z^T(u) W \left[Hx - D_z(u) \right] \geqslant 0. \tag{12}$$

Theorem 1
Consider a nonlinear time-delay system (1) under $d(t) = 0$ satisfying (2), (3), and Assumption 1. Suppose that there exist symmetric matrices X, $\bar{Q}_i$, and $\bar{Z}_j$, for $i = 1, 2, 3$ and $j = 1, 2$, matrices M and G, and a diagonal matrix U, such that the inequalities

$$X > 0,\ U > 0,\ \bar{Q}_i > 0,\ \bar{Z}_j > 0\ \forall i = 1, 2, 3,\ \ j = 1, 2, \tag{13}$$

$$\begin{bmatrix} X & M_{(i)}^T - G_{(i)}^T \\ * & \bar{u}_{(i)}^2 \end{bmatrix} \geqslant 0,\ \ \forall i = 1, \ldots, m, \tag{14}$$

$$\begin{aligned} \bar{\Phi}_1 = \bar{\Phi} - &\begin{bmatrix} 0 & I & 0 & -I & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T \\ &\times \bar{Z}_2 \begin{bmatrix} 0 & I & 0 & -I & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} < 0, \end{aligned} \tag{15}$$

$$\begin{aligned} \bar{\Phi}_2 = \bar{\Phi} - &\begin{bmatrix} 0 & -I & I & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T \\ &\times \bar{Z}_2 \begin{bmatrix} 0 & -I & I & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} < 0, \end{aligned} \tag{16}$$

are satisfied, where

$$\bar{\Phi} = \begin{bmatrix} \bar{\Xi}_1 & A_d X & \bar{Z}_1 & 0 & I & I & -BU + G^T & \tau_1 \bar{\Xi}_3 & \tau_{21} \bar{\Xi}_3 & X\Lambda_f^T & 0 \\ * & \bar{\Xi}_2 & \bar{Z}_2 & \bar{Z}_2 & 0 & 0 & 0 & \tau_1 X A_d^T & \tau_{21} X A_d^T & 0 & X\Lambda_g^T \\ * & * & -\bar{Q}_1 - \bar{Z}_1 - \bar{Z}_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & -\bar{Q}_2 - \bar{Z}_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & -I & 0 & 0 & \tau_1 I & \tau_{21} I & 0 & 0 \\ * & * & * & * & * & -I & 0 & \tau_1 I & \tau_{21} I & 0 & 0 \\ * & * & * & * & * & * & -2U & -\tau_1 U B^T & -\tau_{21} U B^T & 0 & 0 \\ * & * & * & * & * & * & * & -X\bar{Z}_1^{-1}X & 0 & 0 & 0 \\ * & * & * & * & * & * & * & * & -X\bar{Z}_2^{-1}X & 0 & 0 \\ * & * & * & * & * & * & * & * & * & -I & 0 \\ * & * & * & * & * & * & * & * & * & * & -I \end{bmatrix}, \tag{17}$$

$$\begin{aligned} \tau_{21} &= \tau_2 - \tau_1, \\ \bar{\Xi}_1 &= AX + XA^T + BM + M^T B^T + \sum_{i=1}^{3} \bar{Q}_i - \bar{Z}_1, \\ \bar{\Xi}_2 &= -(1-\mu_1)\bar{Q}_3 - 2\bar{Z}_2, \\ \bar{\Xi}_3 &= \left(XA^T + M^T B^T\right). \end{aligned} \tag{18}$$

Then there exists a state feedback controller of the form (8) that guarantees convergence of the state $x(t)$ to the origin for every initial condition belonging to the region $\phi^T(0)P\phi(0) + \Upsilon_1 \phi^T(t)\phi(t) + \Upsilon_2 \dot{\phi}^T(t)\dot{\phi}(t) < 1$ for all $t \in \left[-\tau_2 \quad 0\right]$, where

$$\Upsilon_1 = \tau_1 \lambda_{\max}\left(X^{-1}\bar{Q}_1 X^{-1}\right) + \tau_2 \lambda_{\max}\left(X^{-1}\bar{Q}_2 X^{-1}\right) + \tau_2 \lambda_{\max}\left(X^{-1}\bar{Q}_3 X^{-1}\right) \tag{19}$$

$$\Upsilon_2 = \frac{\tau_1^3}{2} \lambda_{\max}\left(X^{-1}\bar{Z}_1 X^{-1}\right) + \frac{(\tau_1 + \tau_2)\tau_{21}^2}{2} \lambda_{\max}\left(X^{-1}\bar{Z}_2 X^{-1}\right). \tag{20}$$

Further, the closed-loop system state remains bounded by $x^T(t)X^{-1}x(t) < 1$ for all time. The controller gain matrix can then be determined as $F = MX^{-1}$.

Proof

Consider a Lyapunov functional candidate

$$\begin{aligned} V(x,t) = {} & x^T(t)Px(t) + \int_{t-\tau(t)}^{t} x^T(\theta)Q_3 x(\theta)d\theta + \sum_{i=1}^{2} \int_{t-\tau_i}^{t} x^T(\theta)Q_i x(\theta)d\theta \\ & + \tau_1 \int_{-\tau_1}^{0} \int_{t+s}^{t} \dot{x}^T(\theta)Z_1 \dot{x}(\theta)d\theta ds + \tau_{21} \int_{-\tau_2}^{-\tau_1} \int_{t+s}^{t} \dot{x}^T(\theta)Z_2 \dot{x}(\theta)d\theta ds, \end{aligned} \tag{21}$$

where $P = X^{-1}$, $Q_i = X^{-1}\bar{Q}_i X^{-1}$ and $Z_j = X^{-1}\bar{Z}_j X^{-1}$ for $i = 1, 2, 3$ and $j = 1, 2$. Taking the time-derivative of (21) along (10) yields

$$\begin{aligned} \dot{V}(x,t) \leqslant {} & 2x^T(t)P\left((A + BF)x(t) + A_d x(t-\tau(t)) + f(t,x) + g(t,x(t-\tau)) - BDz(u) + d\right) \\ & + \sum_{i=1}^{3} x^T(t)Q_i x(t) - (1-\dot{\tau})x^T(t-\tau(t))Q_3 x(t-\tau(t)) - \sum_{i=1}^{2} x^T(t-\tau_i)Q_i x(t-\tau_i) \\ & + \left((A + BF)x(t) + A_d x(t-\tau(t)) + f(t,x) + g(t,x(t-\tau)) - BDz(u) + d\right)^T \\ & \times \left(\tau_1^2 Z_1 + \tau_{21}^2 Z_2\right)\left((A + BF)x(t) + A_d x(t-\tau(t)) + f(t,x) + g(t,x(t-\tau))\right. \\ & \left. - BDz(u) + d\right) - \tau_1 \int_{t-\tau_1}^{t} \dot{x}^T(\theta)Z_1 \dot{x}(\theta)d\theta - \tau_{21} \int_{t-\tau_2}^{t-\tau_1} \dot{x}^T(\theta)Z_2 \dot{x}(\theta)d\theta. \end{aligned} \tag{22}$$

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Applying (3) and (12) reveals

$$\begin{aligned}\dot{V}(x,t) \leqslant\ & 2x^T(t)P\left((A+BF)x(t) + A_d x(t-\tau(t)) + f(t,x) + g(t,x(t-\tau)) - BDz(u) + d\right) \\ & + \sum_{i=1}^{3} x^T(t)Q_i x(t) - (1-\mu)x^T(t-\tau(t))Q_3 x(t-\tau(t)) - \sum_{i=1}^{2} x^T(t-\tau_i)Q_i x(t-\tau_i) \\ & + \left((A+BF)x(t) + A_d x(t-\tau(t)) + f(t,x) + g(t,x(t-\tau)) - BDz(u) + d\right)^T \\ & \times \left(\tau_1^2 Z_1 + \tau_{21}^2 Z_2\right)\left((A+BF)x(t) + A_d x(t-\tau(t)) + f(t,x) + g(t,x(t-\tau))\right. \\ & \left. - BDz(u) + d\right) - \tau_1 \int_{t-\tau_1}^{t} \dot{x}^T(\theta)Z_1\dot{x}(\theta)d\theta - \tau_{21}\int_{t-\tau_2}^{t-\tau_1} \dot{x}^T(\theta)Z_2\dot{x}(\theta)d\theta \\ & + D_z^T(u)W\left(Hx - D_z(u)\right) + \left(Hx - D_z(u)\right)^T W D_z(u).\end{aligned} \tag{23}$$

For the time, we incorporate (12) to derive the stability condition. However, the validity of (12) depends on the condition provided in (11). Later in the proof, it will be demonstrated that the region (11) remains valid for all the initial conditions belonging to $\phi^T(0)P\phi(0) + \mathrm{T}_1\phi^T(t)\phi(t) + \mathrm{T}_2\dot{\phi}^T(t)\dot{\phi}(t) < 1$ for all $t \in \left[-\tau_2 \quad 0\right]$. Incorporating Assumption 1 for the nonlinear time-delay system (1), we obtain

$$\begin{aligned}\dot{V}(x,t) \leqslant\ & 2x^T(t)P\left((A+BF)x(t) + A_d x(t-\tau(t)) + f(t,x) + g(t,x(t-\tau)) - BDz(u) + d\right) \\ & + \sum_{i=1}^{3} x^T(t)Q_i x(t) - (1-\mu)x^T(t-\tau(t))Q_3 x(t-\tau(t)) - \sum_{i=1}^{2} x^T(t-\tau_i)Q_i x(t-\tau_i) \\ & + \left((A+BF)x(t) + A_d x(t-\tau(t)) + f(t,x) + g(t,x(t-\tau)) - BDz(u) + d\right)^T \\ & \times \left(\tau_1^2 Z_1 + \tau_{21}^2 Z_2\right)\left((A+BF)x(t) + A_d x(t-\tau(t)) + f(t,x) + g(t,x(t-\tau))\right. \\ & \left. - BDz(u) + d\right) - \tau_1 \int_{t-\tau_1}^{t} \dot{x}^T(\theta)Z_1\dot{x}(\theta)d\theta - \tau_{21}\int_{t-\tau_2}^{t-\tau_1} \dot{x}^T(\theta)Z_2\dot{x}(\theta)d\theta \\ & + D_z^T(u)W\left(Hx - D_z(u)\right) + \left(Hx - D_z(u)\right)^T W D_z(u) + f^T(t,x)f(t,x) \\ & - x^T\Lambda_f^T\Lambda_f x + g^T(t,x(t-\tau))g(t,x(t-\tau)) - x^T(t-\tau(t))\Lambda_g^T\Lambda_g x(t-\tau(t)).\end{aligned} \tag{24}$$

Application of Lemma 1 implies that

$$-\tau_1 \int_{t-\tau_1}^{t} \dot{x}^T(\theta)Z_1\dot{x}(\theta)d\theta \leqslant -\left(x(t) - x(t-\tau_1)\right)^T Z_1 \left(x(t) - x(t-\tau_1)\right). \tag{25}$$

It is noted that

$$-\tau_{21}\int_{t-\tau_2}^{t-\tau_1} \dot{x}^T(\theta)Z_2\dot{x}(\theta)d\theta = -\tau_{21}\left[\int_{t-\tau_2}^{t-\tau(t)} \dot{x}^T(\theta)Z_2\dot{x}(\theta)d\theta - \int_{t-\tau(t)}^{t-\tau_1} \dot{x}^T(\theta)Z_2\dot{x}(\theta)d\theta\right]. \tag{26}$$

Applying Lemma 1 (see, for instance, [32]), it is implicit to obtain

$$\begin{aligned} & -\tau_{21}\int_{t-\tau_2}^{t-\tau_1} \dot{x}^T(\theta)Z_2\dot{x}(\theta)d\theta \\ \leqslant\ & -\left(x(t-\tau(t)) - x(t-\tau_2)\right)^T Z_2 \left(x(t-\tau(t)) - x(t-\tau_2)\right) \\ & -\left(x(t-\tau_1) - x(t-\tau(t))\right)^T Z_2 \left(x(t-\tau_1) - x(t-\tau(t))\right) \\ & -\varsigma\left(x(t-\tau(t)) - x(t-\tau_2)\right)^T Z_2 \left(x(t-\tau(t)) - x(t-\tau_2)\right) \\ & -(1-\varsigma)\left(x(t-\tau_1) - x(t-\tau(t))\right)^T Z_2 \left(x(t-\tau_1) - x(t-\tau(t))\right),\end{aligned} \tag{27}$$

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where $\varsigma = (\tau(t) - \tau_1)/(\tau_2 - \tau_1)$. Combining (24), (25), and (27) under $d(t) = 0$ results in

$$\begin{aligned}\dot{V}(x,t) \leqslant\ & x^T\left[(A+BF)^T P + P(A+BF) + \sum_{i=1}^{3} Q_i - Z_1 + \Lambda_f^T \Lambda_f\right] x + 2x^T P A_d x(t-\tau(t)) \\ & + 2x^T Z_1 x(t-\tau_1) + x^T(t-\tau(t))\left[-(1-\mu_1)Q_3 - 2Z_2 + \Lambda_g^T\Lambda_g\right] x(t-\tau(t)) \\ & + 2x^T(t-\tau(t)) Z_2 x(t-\tau_1) + 2x^T(t-\tau(t)) Z_2 x(t-\tau_2) - x^T(t-\tau_1)\left[Q_1 + Z_1 + Z_2\right] \\ & \times x(t-\tau_1) - x^T(t-\tau_2)\left[Q_2 + Z_2\right] x(t-\tau_2) + 2x^T P f(t,x) + 2x^T P g(t,(t-\tau)) \\ & - f^T(t,x) f(t,x) - g^T(t,x(t-\tau)) g(t,x(t-\tau)) + 2D_z^T(u) W H x - 2D_z^T(u) W D_z(u) \\ & - 2x^T PBDz(u) - \varsigma\left(x(t-\tau(t)) - x(t-\tau_2)\right)^T Z_2\left(x(t-\tau(t)) - x(t-\tau_2)\right) \\ & - (1-\varsigma)\left(x(t-\tau_1) - x(t-\tau(t))\right)^T Z_2\left(x(t-\tau_1) - x(t-\tau(t))\right) \\ & + \zeta_1^T\left[\, A+BF \;\; A_d \;\; 0 \;\; 0 \;\; I \;\; I \;\; -B \,\right]^T\left[\tau_1^2 Z_1 + \tau_{21}^2 Z_2\right] \\ & \times\left[\, A+BF \;\; A_d \;\; 0 \;\; 0 \;\; I \;\; I \;\; -B \,\right]\zeta_1,\end{aligned} \tag{28}$$

where

$$\zeta_1^T = \left[\, x^T(t) \;\; x^T(t-\tau(t)) \;\; x^T(t-\tau_1) \;\; x^T(t-\tau_2) \;\; f^T(t,x) \;\; g^T(t,x(t-\tau)) \;\; Dz^T(u) \,\right]. \tag{29}$$

From (28) and (29), we have

$$\begin{aligned}\dot{V}(x,t) \leqslant\ & \zeta_1^T \Upsilon \zeta_1 - \varsigma\left(x(t-\tau(t)) - x(t-\tau_2)\right)^T Z_2\left(x(t-\tau(t)) - x(t-\tau_2)\right) \\ & - (1-\varsigma)\left(x(t-\tau_1) - x(t-\tau(t))\right)^T Z_2\left(x(t-\tau_1) - x(t-\tau(t))\right),\end{aligned} \tag{30}$$

$$\dot{V}(x,t) \leqslant \zeta_1^T\left[\varsigma\Upsilon_1 + (1-\varsigma)\Upsilon_2\right]\zeta_1, \tag{31}$$

where

$$\Upsilon_1 = \Upsilon - \left[\, 0 \;\; I \;\; 0 \;\; -I \;\; 0 \;\; 0 \;\; 0 \,\right]^T Z_2\left[\, 0 \;\; I \;\; 0 \;\; -I \;\; 0 \;\; 0 \;\; 0 \,\right], \tag{32}$$

$$\Upsilon_2 = \Upsilon - \left[\, 0 \;\; -I \;\; I \;\; 0 \;\; 0 \;\; 0 \;\; 0 \,\right]^T Z_2\left[\, 0 \;\; -I \;\; I \;\; 0 \;\; 0 \;\; 0 \;\; 0 \,\right], \tag{33}$$

$$\begin{aligned}\Upsilon =\ & \begin{bmatrix} \Xi_1 + \Lambda_f^T\Lambda_f & PA_d & Z_1 & 0 & P & P & -PB + H^T W \\ * & \Xi_2 + \Lambda_g^T\Lambda_g & Z_2 & Z_2 & 0 & 0 & 0 \\ * & * & -Q_1 - Z_1 - Z_2 & 0 & 0 & 0 & 0 \\ * & * & * & -Q_2 - Z_2 & 0 & 0 & 0 \\ * & * & * & * & -I & 0 & 0 \\ * & * & * & * & * & -I & 0 \\ * & * & * & * & * & * & -2W \end{bmatrix} \\ & + \left[\, \Xi_3 \;\; A_d \;\; 0 \;\; 0 \;\; I \;\; I \;\; -B \,\right]^T\left[\tau_1^2 Z_1 + \tau_{21}^2 Z_2\right] \\ & \times\left[\, \Xi_3 \;\; A_d \;\; 0 \;\; 0 \;\; I \;\; I \;\; -B \,\right],\end{aligned} \tag{34}$$

$$\begin{aligned}\Xi_1 &= (A+BF)^T P + P(A+BF) + \sum_{i=1}^{3} Q_i - Z_1, \\ \Xi_2 &= -(1-\mu_1)Q_3 - 2Z_2, \\ \Xi_3 &= A + BF.\end{aligned} \tag{35}$$

As $0 < \varsigma < 1$, the inequalities $\Upsilon_1 < 0$ and $\Upsilon_2 < 0$ ensure that $\dot{V}(x,t) < 0$; that is, the state of the closed-loop system asymptotically converges to the origin. The application of the Schur complement to $\Upsilon_1 < 0$ and $\Upsilon_2 < 0$ yields

$$\begin{aligned}\Phi_1 = \Phi &- \left[\, 0 \;\; I \;\; 0 \;\; -I \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \,\right]^T \\ & \times Z_2\left[\, 0 \;\; I \;\; 0 \;\; -I \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \,\right] < 0,\end{aligned} \tag{36}$$

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$$\begin{aligned}\Phi_2 = \Phi - \begin{bmatrix} 0 & -I & I & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T \\ \times Z_2 \begin{bmatrix} 0 & -I & I & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} < 0,\end{aligned} \tag{37}$$

$$\Phi = \begin{bmatrix} \Xi_1 & PA_d & Z_1 & 0 & P & P & -PB + H^T W & \tau_1 \Xi_3^T & \tau_{21} \Xi_3^T & \Lambda_f^T & 0 \\ * & \Xi_2 & Z_2 & Z_2 & 0 & 0 & 0 & \tau_1 A_d^T & \tau_{21} A_d^T & 0 & \Lambda_g^T \\ * & * & -Q_1 - Z_1 - Z_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & -Q_2 - Z_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & -I & 0 & 0 & \tau_1 I & \tau_{21} I & 0 & 0 \\ * & * & * & * & * & -I & 0 & \tau_1 I & \tau_{21} I & 0 & 0 \\ * & * & * & * & * & * & -2W & -\tau_1 B^T & -\tau_{21} B^T & 0 & 0 \\ * & * & * & * & * & * & * & -Z_1^{-1} & 0 & 0 & 0 \\ * & * & * & * & * & * & * & * & -Z_2^{-1} & 0 & 0 \\ * & * & * & * & * & * & * & * & * & -I & 0 \\ * & * & * & * & * & * & * & * & * & * & -I \end{bmatrix}. \tag{38}$$

Inequalities (15) and (16) are obtained by application of the congruence transform by pre-multiplying and post-multiplying the diagonal structure $diag(\varphi_1, \varphi_2, \varphi_3, \varphi_4)$ to the inequalities (36) and (37), where $\varphi_1 = diag(P^{-1}, P^{-1}, P^{-1}, P^{-1})$, $\varphi_2 = diag(I, I, W^{-1})$, $\varphi_3 = diag(P^{-1}, P^{-1})$, and $\varphi_4 = diag(I, I)$, and further, by substituting $X = P^{-1}U = W^{-1}, G = HX, M = FX\bar{Q}_i = XQ_iX$ and $\bar{Z}_j = XZ_jX$ for $i = 1, 2, 3$ and $j = 1, 2$.

Given the Lyapunov functional (21), the condition $\dot{V}(x,t) < 0$ implies that $V(x(t),t) < V(x,0)\ \forall t > 0$, which further affords $x^T(t)X^{-1}x(t) < V(x,0), \forall t > 0$. From (21), we obtain

$$\begin{aligned} V(x,0) = x^T(0)Px(0) + \int_{-\tau(t)}^{0} x^T(\theta)Q_3x(\theta)d\theta + \sum_{i=1}^{2}\int_{-\tau_i}^{0} x^T(\theta)Q_ix(\theta)d\theta \\ + \tau_1\int_{-\tau_1}^{0}\int_{s}^{0}\dot{x}^T(\theta)Z_1\dot{x}(\theta)d\theta ds + \tau_{21}\int_{-\tau_2}^{-\tau_1}\int_{s}^{0}\dot{x}^T(\theta)Z_2\dot{x}(\theta)d\theta ds. \end{aligned} \tag{39}$$

It is noted that

$$\int_{-\tau(t)}^{0} x^T(\theta)Q_3x(\theta)d\theta + \sum_{i=1}^{2}\int_{-\tau_i^0 x^T}(\theta)Q_ix(\theta)d\theta \leqslant T_1\phi^T(t)\phi(t), \tag{40}$$

$$\tau_1\int_{-\tau_1}^{0}\int_{s}^{0}\dot{x}^T(\theta)Z_1\dot{x}(\theta)d\theta ds + \tau_{21}\int_{-\tau_2}^{-\tau_1}\int_{s}^{0}\dot{x}^T(\theta)Z_2\dot{x}(\theta)d\theta ds \leqslant T_2\dot{\phi}^T(t)\dot{\phi}(t). \tag{41}$$

Hence, the inequality $V(x,0) < 1$ holds for any initial condition satisfying $\phi^T(0)P\phi(0) + T_1\phi^T(t)\phi(t) + T_2\dot{\phi}^T(t)\dot{\phi}(t) < 1$ for all $t \in \begin{bmatrix} -\tau_2 & 0 \end{bmatrix}$, which along with the inequality $x^T(t)X^{-1}x(t) < V(x,0)$, implies that $x^T(t)X^{-1}x(t) < 1 \forall t > 0$. By including the region $x^T(t)X^{-1}x(t) < 1$ in $S(\bar{u})$, we have

$$\begin{bmatrix} X^{-1} & F_{(i)}^T - H_{(i)}^T \\ * & \bar{u}_{(i)}^2 \end{bmatrix} \geqslant 0, \quad i = 1, \ldots, m. \tag{42}$$

Consequently, the region (11) and the local sector condition (12) remain valid. LMI (14) is obtained by employing the congruence transform, $diag(X, I)$, and substituting $G = HX$ and $M = FX$, which completes the proof of Theorem 1. □

Remark 1

A sufficient condition for stabilization of nonlinear time-delay systems under actuator saturation constraint was provided in the form of Theorem 1. It should be noted that data on the control of non-linear time-delay systems subject to input saturation are lacking in the literature. It is noteworthy that the conditions in Theorem 1 were established by the delay-range-dependent approach in contrast to the traditional work on linear systems (e.g., [33–36]). Note that the delay-range-dependent control methodologies are shown to be less conservative (particularly, for interval delays with a non-zero lower bound) than the conventional delay-dependent and delay-independent approaches (see, for instance, [20–26]). Additionally, another distinct feature of the proposed state feedback control treatment, in contrast to other techniques (e.g., [33–35]), is its applicability to input-constrained nonlinear systems in the presence of time-varying delays.

A delay-range-dependent approach for stabilization of a linear system, by substituting $f(t,x) = 0$ and $g(t, x(t-\tau)) = 0$ in (1), can be derived in a way similar to that of Theorem 1. The following corollary provides the corresponding results for linear systems.

Corollary 1
Consider a nonlinear time-delay system (1) satisfying (2) and (3) $f(t,x) = 0, g(t, x(t-\tau)) = 0$ and $d = 0$. Suppose that there exist symmetric matrices X, $\bar{Q}_i$, and $\bar{Z}_j$, for $i = 1,2,3$ and $j = 1,2$, matrices M and G, and a diagonal matrix U, such that the inequalities (13) and (14) and

$$\bar{\Phi}_{C1} - \begin{bmatrix} 0 & I & 0 & -I & 0 & 0 & 0 \end{bmatrix}^T \bar{Z}_2 \begin{bmatrix} 0 & I & 0 & -I & 0 & 0 & 0 \end{bmatrix} < 0 \tag{43}$$

$$\bar{\Phi}_{C1} - \begin{bmatrix} 0 & -I & I & 0 & 0 & 0 & 0 \end{bmatrix}^T \bar{Z}_2 \begin{bmatrix} 0 & -I & I & 0 & 0 & 0 & 0 \end{bmatrix} < 0 \tag{44}$$

are satisfied, where

$$\bar{\Phi}_{C1} = \begin{bmatrix} \bar{\Xi}_1 & A_d X & \bar{Z}_1 & 0 & -BU + G^T & \tau_1 \bar{\Xi}_3 & \tau_{21} \bar{\Xi}_3 \\ * & \bar{\Xi}_2 & \bar{Z}_2 & \bar{Z}_2 & 0 & \tau_1 X A_d^T & \tau_{21} X A_d^T \\ * & * & -\bar{Q}_1 - \bar{Z}_1 - \bar{Z}_2 & 0 & 0 & 0 & 0 \\ * & * & * & -\bar{Q}_2 - \bar{Z}_2 & 0 & 0 & 0 \\ * & * & * & * & -2U & -\tau_1 U B^T & -\tau_{21} U B^T \\ * & * & * & * & * & -X \bar{Z}_1^{-1} X & 0 \\ * & * & * & * & * & * & -X \bar{Z}_2^{-1} X \end{bmatrix}. \tag{45}$$

Then there exists a state feedback controller of form (8) that guarantees convergence of the state $x(t)$ to the origin for every initial condition belonging to the region $\phi^T(0) P \phi(0) + \mathrm{T}_1 \phi^T(t)\phi(t) + \mathrm{T}_2 \dot{\phi}^T(t)\dot{\phi}(t) < 1$ for all $t \in \begin{bmatrix} -\tau_2 & 0 \end{bmatrix}$. Further, the closed-loop system state remains bounded by $x^T(t) X^{-1} x(t) < 1$ for all time. The controller gain matrix therefore can be determined as $F = M X^{-1}$.

Remark 2
Corollary 1 provides a specific case of Theorem 1 for the linear time-delay systems. The delay-range-dependent control strategy for linear systems subject to input saturation has not been thoroughly addressed in the literature except some interesting work such as [20]. Note that the results in Corollary 1 further elaborate the effectiveness of the proposed approach in Theorem 1. In contrast to [20], the condition provided in Corollary 1 avoids a free-weighting-matrix approach leading to simple and straightforward results. The proposed approach in Corollary 1 studies interval delays within a range unlike the traditional control approaches for linear time-delay systems under input saturation.

The approach offered in Theorem 1 is applicable to time-delay systems with interval time-varying delays starting from a non-zero value. The delay-dependent results, provided in the following corollary, are derived as a specific case of Theorem 1 by setting $Q_1 = 0$ and $Z_1 = 0$.

Corollary 2
Consider a nonlinear time-delay system (1) under $d(t) = 0$ satisfying $\tau_1 = 0$, (2), (3), and Assumption 1. Suppose that there exist symmetric matrices X, $\bar{Q}_i$, and $\bar{Z}_2$, for $i = 2,3$, matrices M and G, and a diagonal matrix U, such that the inequalities (14)

$$X > 0, \;\; U > 0, \;\; \bar{Q}_i > 0, \;\; \bar{Z}_2 > 0 \;\forall i = 2,3, \tag{46}$$

$$\begin{aligned} \bar{\Phi}_{C2} &- \begin{bmatrix} 0 & I & 0 & -I & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T \\ &\times \bar{Z}_2 \begin{bmatrix} 0 & I & 0 & -I & 0 & 0 & 0 & 0 & 0 \end{bmatrix} < 0, \end{aligned} \tag{47}$$

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$$\bar{\Phi}_{C2} - \begin{bmatrix} 0 & -I & I & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T \times \bar{Z}_2 \begin{bmatrix} 0 & -I & I & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} < 0, \tag{48}$$

are satisfied, where

$$\bar{\Phi}_{C2} = \begin{bmatrix} \bar{\Psi}_1 & A_d X + \bar{Z}_2 & 0 & I & I & -BU + G^T & \tau_2 \bar{\Xi}_3 & X\Lambda_f^T & 0 \\ * & \bar{\Psi}_2 & \bar{Z}_2 & 0 & 0 & 0 & \tau_{21} X A_d^T & 0 & X\Lambda_g^T \\ * & * & -\bar{Q}_2 - \bar{Z}_2 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & -I & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & -I & 0 & 0 & 0 & 0 \\ * & * & * & * & * & -2U & -\tau_2 U B^T & 0 & 0 \\ * & * & * & * & * & * & -X\bar{Z}_2^{-1}X & 0 & 0 \\ * & * & * & * & * & * & * & -I & 0 \\ * & * & * & * & * & * & * & * & -I \end{bmatrix}, \tag{49}$$

$$\bar{\Psi}_1 = AX + XA^T + BM + M^T B^T + \sum_{i=2}^{3} \bar{Q}_i - \bar{Z}_2, \qquad \bar{\Psi}_2 = -(1-\mu_1)\bar{Q}_3 - 2\bar{Z}_2. \tag{50}$$

Then there exists a state feedback controller of form (8) that guarantees convergence of the state $x(t)$ to the origin for every initial condition belonging to the region $\phi^T(0)P\phi(0) + \mathrm{T}_3\phi^T(t)\phi(t) + \mathrm{T}_4\dot{\phi}^T(t)\dot{\phi}(t) < 1$ for all $t \in \begin{bmatrix} -\tau_2 & 0 \end{bmatrix}$, where

$$\mathrm{T}_3 = \tau_2 \lambda_{\max}\left(X^{-1}\bar{Q}_2 X^{-1}\right) + \tau_2 \lambda_{\max}\left(X^{-1}\bar{Q}_3 X^{-1}\right) \tag{51}$$

$$\mathrm{T}_4 = \frac{\tau_2^3}{2}\lambda_{\max}\left(X^{-1}\bar{Z}_2 X^{-1}\right). \tag{52}$$

Further, the closed-loop system state remains bounded by $x^T(t)X^{-1}x(t) < 1$ for all time. The controller gain matrix therefore can be determined as $F = MX^{-1}$.

Theorem 1 provides the delay-range-dependent condition for controller design by incorporating the bound on the time-delay derivative. The results of Theorem 1 can be extended to the case where an *a priori* delay-derivative bound is unidentified. By setting $Q_3 = 0$, the following corresponding delay-rate-independent controller synthesis condition is provided.

Corollary 3

Consider a nonlinear time-delay system (1) under $d(t) = 0$ satisfying (2) and Assumption 1. Suppose that there exist symmetric matrices X, $\bar{Q}_i$, and $\bar{Z}_j$, for $i = 1, 2$ and $j = 1, 2$, matrices M and G, and a diagonal matrix U, such that the inequalities (14)

$$X > 0, \;\; U > 0, \;\; \bar{Q}_i > 0, \;\; \bar{Z}_j > 0 \, \forall i = 1, 2, \;\; j = 1, 2, \tag{53}$$

$$\bar{\Phi}_{C3} - \begin{bmatrix} 0 & I & 0 & -I & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T \times \bar{Z}_2 \begin{bmatrix} 0 & I & 0 & -I & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} < 0, \tag{54}$$

$$\bar{\Phi}_{C3} - \begin{bmatrix} 0 & -I & I & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T \times \bar{Z}_2 \begin{bmatrix} 0 & -I & I & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} < 0, \tag{55}$$

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are satisfied, where

$$\bar{\Phi}_{C3}=\begin{bmatrix} \bar{\Pi}_1 & A_dX & \bar{Z}_1 & 0 & I & I & -BU+G^T & \tau_1\bar{\Xi}_3 & \tau_{21}\bar{\Xi}_3 & X\Lambda_f^T & 0 \\ * & -2\bar{Z}_2 & \bar{Z}_2 & \bar{Z}_2 & 0 & 0 & 0 & \tau_1XA_d^T & \tau_{21}XA_d^T & 0 & X\Lambda_g^T \\ * & * & -\bar{Q}_1-\bar{Z}_1-\bar{Z}_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & -\bar{Q}_2-\bar{Z}_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & -I & 0 & 0 & \tau_1 I & \tau_{21}I & 0 & 0 \\ * & * & * & * & * & -I & 0 & \tau_1 I & \tau_{21}I & 0 & 0 \\ * & * & * & * & * & * & -2U & -\tau_1B^T & -\tau_{21}B^T & 0 & 0 \\ * & * & * & * & * & * & * & -X\bar{Z}_1^{-1}X & 0 & 0 & 0 \\ * & * & * & * & * & * & * & * & -X\bar{Z}_2^{-1}X & 0 & 0 \\ * & * & * & * & * & * & * & * & * & -I & 0 \\ * & * & * & * & * & * & * & * & * & * & -I \end{bmatrix}, \tag{56}$$

$$\bar{\Pi}_1 = AX + XA^T + BM + M^TB^T + \sum_{i=1}^{2}\bar{Q}_i - \bar{Z}_1. \tag{57}$$

Then there exists a state feedback controller of the form (8) that guarantees convergence of the state $x(t)$ to the origin for every initial condition belonging to the region $\phi^T(0)P\phi(0) + \mathrm{T}_5\phi^T(t)\phi(t) + \mathrm{T}_2\dot{\phi}^T(t)\dot{\phi}(t) < 1$ for all $t \in \left[-\tau_2 \quad 0\right]$, where

$$\mathrm{T}_5 = \tau_1\lambda_{\max}\left(X^{-1}\bar{Q}_1X^{-1}\right) + \tau_2\lambda_{\max}\left(X^{-1}\bar{Q}_2X^{-1}\right) \tag{58}$$

Further, the closed-loop system state remains bounded by $x^T(t)X^{-1}x(t) < 1$ for all time. The controller gain matrix therefore can be determined as $F = MX^{-1}$.

Remark 3
Delay-dependent controller design approaches for systems containing nonlinearities associated with both the input and the plant state remain uncommon in the literature. Corollary 2 shows that the delay-dependent controller synthesis condition can be derived as a particular case of the proposed delay-range-dependent approach in Theorem 1. Hence, the new controller synthesis solution in Corollary 2, capable of achieving the delay-dependent stability, is obtained as a specific scenario of the main results in Theorem 1. It is notable that the controller synthesis condition in Theorem 1 cannot be applied to the input-constrained nonlinear time-delay systems with varying time-lags of an unknown delay-derivative bound. This particular controller synthesis approach was addressed in Corollary 3 to provide a comprehensive and widely applicable solution to the problem.

Stabilization of a nonlinear time-delay system (1) subjected to input saturation under various conditions has been addressed in Theorem 1 and Corollaries 1, 2, and 3 by assuming $d(t) = 0$. Because of the complexity arising in the analysis of the region of stability, formulation of a robust controller to cope with the input-constrained delayed nonlinear systems against disturbance is a non-trivial problem. However, from the practical application point of view, it will be interesting to regard both the input saturation and the disturbance because of the bounded input limitation of actuators and the unwanted effects of the surrounding environment, respectively. A local controller synthesis condition, ensuring L_2 gain reduction from disturbance d to output y, is provided in Theorem 2.

Theorem 2
Consider a nonlinear time-delay system (1) satisfying (2), (3), and Assumption 1. Suppose that there exist symmetric matrices X, $\bar{Q}_i$, and $\bar{Z}_j$, for $i = 1, 2, 3$ and $j = 1, 2$, matrices M and G, a diagonal matrix U, and scalars λ, σ, and ξ such that the inequalities (13),

$$\lambda > 0, \;\; \sigma > 0, \;\; \xi > 0, \tag{59}$$

$$\begin{bmatrix} X & M_{(i)}^T - G_{(i)}^T \\ * & \xi\bar{u}_{(i)}^2 \end{bmatrix} \geqslant 0, \;\; \forall i = 1, \ldots, m, \tag{60}$$

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$$\begin{aligned}\bar{\Theta}_1 = \bar{\Theta} - &\begin{bmatrix} 0 & I & 0 & -I & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T \\ &\times \bar{Z}_2 \begin{bmatrix} 0 & I & 0 & -I & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} < 0, \end{aligned} \tag{61}$$

$$\begin{aligned}\bar{\Theta}_1 = \bar{\Theta} - &\begin{bmatrix} 0 & -I & I & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T \\ &\times \bar{Z}_2 \begin{bmatrix} 0 & -I & I & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} < 0, \end{aligned} \tag{62}$$

are satisfied, where

$$\bar{\Theta} = \begin{bmatrix}
\bar{\Xi}_1 & A_d X & \bar{Z}_1 & 0 & X & X & -BU + G^T & I & XC^T & \tau_1 \bar{\Xi}_3 & \tau_{21} \bar{\Xi}_3 & X\Lambda_f^T & 0 \\
* & \bar{\Xi}_2 & \bar{Z}_2 & \bar{Z}_2 & 0 & 0 & 0 & 0 & 0 & \tau_1 X A_d^T & \tau_{21} X A_d^T & 0 & X\Lambda_g^T \\
* & * & -\bar{Q}_1 - \bar{Z}_1 - \bar{Z}_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & -\bar{Q}_2 - \bar{Z}_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & -I & 0 & 0 & 0 & 0 & \tau_1 I & \tau_{21} I & 0 & 0 \\
* & * & * & * & * & -I & 0 & 0 & 0 & \tau_1 I & \tau_{21} I & 0 & 0 \\
* & * & * & * & * & * & -2U & 0 & 0 & -\tau_1 B^T & -\tau_{21} B^T & 0 & 0 \\
* & * & * & * & * & * & * & -\sigma I & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & * & * & * & * & -\lambda I & 0 & 0 & 0 & 0 \\
* & * & * & * & * & * & * & * & * & -X\bar{Z}_1^{-1}X & 0 & 0 & 0 \\
* & * & * & * & * & * & * & * & * & * & -X\bar{Z}_2^{-1}X & 0 & 0 \\
* & * & * & * & * & * & * & * & * & * & * & -I & 0 \\
* & * & * & * & * & * & * & * & * & * & * & * & -I
\end{bmatrix} \tag{63}$$

and $\delta^{-1} = (\sigma\xi)^{-1}$ is the maximum limit on the L_2 norm of disturbance, satisfying $\|d\|_2^2 < \delta^{-1}$. Then there exists a state feedback controller of form (8) that guarantees the following:

(i) Asymptotic convergence of the state $x(t)$ to the origin under every initial condition belonging to the region $\phi^T(0)P\phi(0) + \mathrm{T}_1\phi^T(t)\phi(t) + \mathrm{T}_2\dot{\phi}^T(t)\dot{\phi}(t) < 1$ for all $t \in \begin{bmatrix} -\tau_2 & 0 \end{bmatrix}$, if $d(t) = 0$.

(ii) A bounded L_2 gain, by a scalar $\gamma = \sqrt{\lambda\sigma}$, from the disturbance $d(t)$ to the output $y(t)$, if $d(t) \neq 0$.

(iii) And bounded closed-loop system state by $x^T(t)\xi X^{-1}x(t) < 1$ for any initial condition satisfying $\phi^T(0)P\phi(0) + \mathrm{T}_1\phi^T(t)\phi(t) + \mathrm{T}_2\dot{\phi}^T(t)\dot{\phi}(t) < 1$, $\forall t \in \begin{bmatrix} -\tau_2 & 0 \end{bmatrix}$, and for all the L_2 norm bounded disturbances.

The controller gain matrix therefore can be determined as $F = MX^{-1}$.

Proof

Consider the aforementioned Lyapunov functional (21) and the inequality given by

$$\dot{V}(x,t) + \lambda^{-1}y^T(t)y(t) - \sigma d^T(t)d(t) < 0. \tag{64}$$

By integrating (64) from 0 to T, we obtain

$$V(x,T) - V(x,0) + \lambda^{-1}\int_0^T y^T(t)y(t)dt - \sigma\int_0^T d^T(t)d(t)dt < 0. \tag{65}$$

From (21), (64), and (65), the following are implied:

(a) If $d = 0$, $\dot{V} < 0$, that is, the closed-loop system is asymptotically stable for all initial conditions verifying $\phi^T(0)P\phi(0) + \mathrm{T}_1\phi^T(t)\phi(t) + \mathrm{T}_2\dot{\phi}^T(t)\dot{\phi}(t) < 1$. Moreover, the inequality (64) implies that the L_2 gain from the disturbance $d(t)$ to the output $y(t)$ remains bounded by $\gamma = \sqrt{\lambda\sigma}$, if $d \neq 0$.

(b) And given that $\|d\|_2^2 < \delta^{-1}$, (65) entails $V(t,e) - V(0,e) < \sigma\delta^{-1}$, which along with (39) and $\phi^T(0)P\phi(0) + \mathrm{T}_1\phi^T(t)\phi(t) + \mathrm{T}_2\dot{\phi}^T(t)\dot{\phi}(t) < 1$ implies that the closed-loop system state does not leave the ellipsoidal region $x^T(t)\xi X^{-1}x(t) < 1$ for all time, where $\xi = \sigma^{-1}\delta$.

By including the ellipsoidal region $x^T(t)\xi X^{-1}x(t) < 1$ into (11), LMI (60) is obtained. Utilizing (12), (24), (25), (27), Assumption 1, and $\dot{V}(x,t) + \lambda^{-1}y^T y - \sigma d^T d < 0$ in the same way as in the proof of Theorem 1, we obtain

$$\begin{aligned}\Theta_1 = \Theta - &\begin{bmatrix} 0 & I & 0 & -I & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T \\ &\times Z_2 \begin{bmatrix} 0 & I & 0 & -I & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} < 0,\end{aligned} \tag{66}$$

$$\begin{aligned}\Theta_2 = \Theta - &\begin{bmatrix} 0 & -I & I & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T \\ &\times Z_2 \begin{bmatrix} 0 & -I & I & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} < 0,\end{aligned} \tag{67}$$

$$\Theta = \begin{bmatrix} \Xi_1 & PA_d & Z_1 & 0 & P & P & -PB + H^T W & P & C^T & \tau_1 \Xi_3^T & \tau_{21}\Xi_3^T & \Lambda_f^T & 0 \\ * & \Xi_2 & Z_2 & Z_2 & 0 & 0 & 0 & 0 & 0 & \tau_1 A_d^T & \tau_{21} A_d^T & 0 & \Lambda_g^T \\ * & * & -Q_1 - Z_1 - Z_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & -Q_2 - Z_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & -I & 0 & 0 & 0 & 0 & \tau_1 I & \tau_{21} I & 0 & 0 \\ * & * & * & * & * & -I & 0 & 0 & 0 & \tau_1 I & \tau_{21} I & 0 & 0 \\ * & * & * & * & * & * & -2W & 0 & 0 & -\tau_1 B^T & -\tau_{21} B^T & 0 & 0 \\ * & * & * & * & * & * & * & -\sigma I & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & * & -\lambda I & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & * & * & -Z_1^{-1} & 0 & 0 & 0 \\ * & * & * & * & * & * & * & * & * & * & -Z_2^{-1} & 0 & 0 \\ * & * & * & * & * & * & * & * & * & * & * & -I & 0 \\ * & * & * & * & * & * & * & * & * & * & * & * & -I \end{bmatrix}. \tag{68}$$

Pre-multiplying and post-multiplying $diag(\varphi_1, \varphi_2, \varphi_3, \varphi_4)$ to the inequalities (66) and (67), (where $\varphi_1 = diag(P^{-1}, P^{-1}, P^{-1}, P^{-1}), \varphi_2 = diag(I, I, W^{-1}, I, I), \varphi_3 = diag(P^{-1}, P^{-1})$, and $\varphi_4 = diag(I, I)$) using (68), and employing the transformations $X = P^{-1}, U = W^{-1}, G = HX, M = FX, \bar{Q}_i = XQ_iX$ and $\bar{Z}_j = XZ_jX$ (with $i = 1, 2, 3$ and $j = 1, 2$), we obtain (61)–(63), which ends the proof of Theorem 2. □

Remark 4

In contrast to the traditional L_2 gain reduction approaches for systems under input saturation (see, for instance, [27, 29, 30] and references therein), two constants λ and σ are exploited in (64) for derivation of the robust controller synthesis condition in Theorem 2. Utilization of the two constants in the inequality can facilitate obtainment of a feasible solution through the constraints in Theorem 2. The results provided in Theorem 2 are practicable for the formulation of a robust controller against perturbation and undesirable disturbance consequences as well as varying time delays, actuator saturation, and dynamical nonlinearities. Some delay-range-dependent results like [22] have been reported for global H_∞ controller synthesis of linear systems in the absence of input saturation. It should be noted that the present methodology for local robust delay-range-dependent controller synthesis is not a simple extension of the traditionalistic global L_2 gain reduction-based controller design treatments.

A problem of concern for the delay-range-dependent schemes is the difficulty of handling the controller synthesis conditions through the convex feasibility paradigm. To deal with this dilemma, the non-convex constraints in Corollaries 1–3 and Theorems 1 and 2 can be converted to convex constraints with a nonlinear objective function. For instance, the conditions in Theorem 2 are converted to

$$\begin{aligned} \min\ \text{trace}\left(XP + Y_1R_1 + Y_2R_2 + \bar{Z}_1S_1 + \bar{Z}_2S_2\right) \\ \text{subject to } (13), (59), (60), (61)^*, (62)^*, \\ \begin{bmatrix} P & I \\ * & X \end{bmatrix} \geqslant 0,\ \begin{bmatrix} Y_i & I \\ * & R_i \end{bmatrix} \geqslant 0,\ \begin{bmatrix} \bar{Z}_i & I \\ * & S_i \end{bmatrix} \geqslant 0,\ , i = 1, 2, \end{aligned} \tag{69}$$

where (61)* and (62)* represent (61) and (62), respectively, by substituting $Y_1 = X\bar{Z}_1^{-1}X$ and $Y_2 = X\bar{Z}_2^{-1}X$. The optimization problem (69) can be straightforwardly resolved by application of a recursive approach based on the cone complementary linearization algorithm and the convex LMI-routines (see [24, 37] and references therein).

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Remark 5
Control of nonlinear systems under input saturation is an inherently challenging research problem, which has not been frequently studied owing to the involvement of both input and state nonlinearities in contrast to the linear systems. To address these nonlinearities, some recent studies like [29, 30] have incorporated the local sector condition and bounds on the nonlinear function to ensure a region of stability. However, stabilization of the input-constrained nonlinear time-delay systems introduces much more difficulty in designing a controller because of the additional complication in determining an estimate of the region of stability by complex forms of Lyapunov–Krasovskii functional, especially, for the interval time-delays of time-varying nature. Another major concern while formulating controller synthesis conditions for such non-trivial control problems is to establish convex constraints for seeking the controller gains on account of various restrictions of state nonlinearities, input saturation, and time-delays. It is worth mentioning that the present study addressed this non-trivial control problem by employing various tools and bounds like the delay-interval information, delay-derivative bound knowledge, limits on nonlinear functions, Lyapunov–Krasovskii functional, ellipsoidal analysis, regional stability, L_2 stability, local sector condition, Jensen's inequality, and cone complementary linearization algorithm.

Remark 6
The present work on stabilization controller synthesis for the nonlinear time-delay systems under input saturation has been established on the model-based control theory. However, in industrial control applications for large-scale processes, sometimes it is almost impossible to develop a reliable controller because of the difficulty in obtaining a trustworthy plant model [38, 39]. In such situations, data-driven methods (based on fault detection, isolation, and fault-tolerant control, which gains considerable research attention nowadays) can be employed to attain safety and reliability of the costly and complicated industrial processes (see details in performance-oriented results for vehicle suspension and industrial benchmark systems in [40, 41]). To achieve advantages of the data-driven approaches, fault-tolerant control of nonlinear time-delay systems, with delays belonging to a range and under actuator faults like saturation, can be considered as a future research direction.

4. SIMULATION RESULTS

Two numerical examples of an unstable system and an industrial large-scale chemical reactor are provided in this section.

Example 1
The proposed control scheme is tested for an unstable nonlinear time-delay network [42], given by

$$A = \begin{bmatrix} 1 & 0.1 \\ 0 & 1 \end{bmatrix},\ A_d = \begin{bmatrix} -0.1 & 0.01 \\ 0 & -0.1 \end{bmatrix},\ B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},\ C = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}, \tag{70}$$

$$\bar{u} = \begin{bmatrix} 5 \\ 5 \end{bmatrix},\ f(x) = \begin{bmatrix} \tanh x_1 - 0.1 \tanh x_2 \\ -0.5 \tanh x_1 + 0.15 \tanh x_2 \end{bmatrix}, \tag{71}$$

$$g(x(t-\tau)) = \begin{bmatrix} -0.24 \tanh x_1(t-\tau) - 0.02 \tanh x_2(t-\tau) \\ -0.2 \tanh x_1(t-\tau) - 0.1 \tanh x_2(t-\tau) \end{bmatrix}. \tag{72}$$

The controller parameters are obtained as

$$F = \begin{bmatrix} -2.721 & -0.10 \\ -0.0001 & -2.721 \end{bmatrix},\ X = \begin{bmatrix} 1.202 & -0.0004 \\ -0.0004 & 1.202 \end{bmatrix}, \tag{73}$$

and $\gamma = 0.95$ by solving Theorem 2 for $\tau_1 = 0.7, \tau_2 = 1, \mu = 0.1, \Lambda_f = diag(1,1)$ and $\Lambda_g = diag(0.3, 0.3)$. The open-loop response of the system described by (70)–(72) is demonstrated in Figure 1 for

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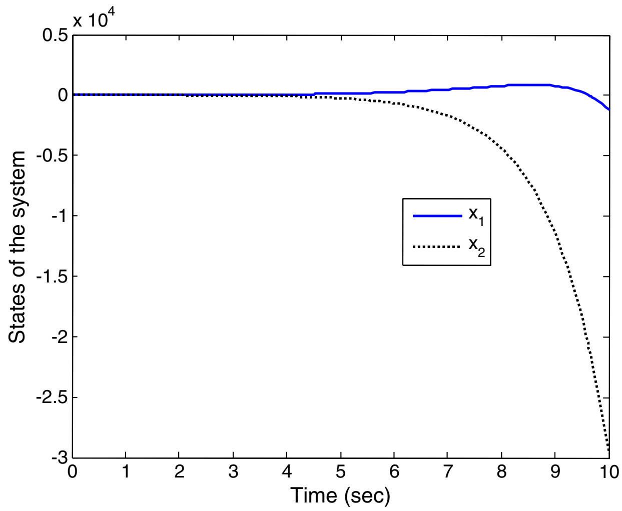

Figure 1. Open-loop response of the unstable system.

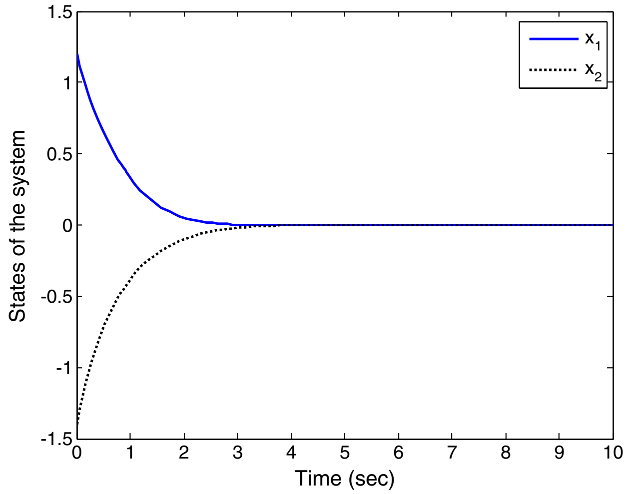

Figure 2. Closed-loop response under input saturation, showing convergence of system's states to zero.

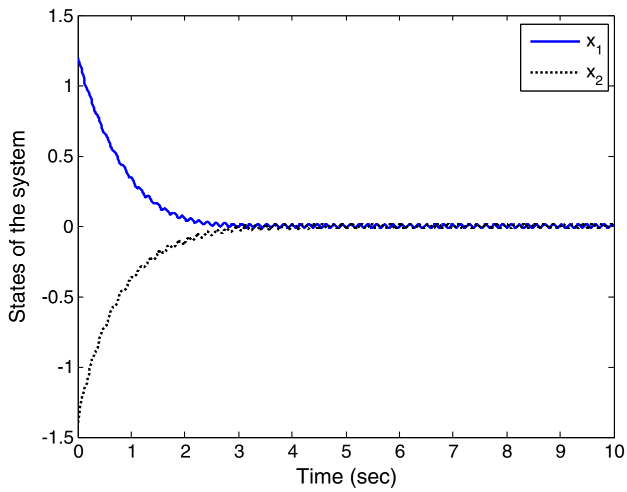

Figure 3. Robustness of the proposed control scheme against disturbance.

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$$\tau(t) = t - 0.8 - 0.1 \sin t. \tag{74}$$

It can be verified that the system is highly unstable. The closed-loop system response using the proposed controller for the gain matrix provided in (67) is shown in Figure 2. Note that by application of the proposed control action, the system's states converge to zero, which serves to demonstrate that the proposed control approach can ensure delay-range-dependent asymptotic stability of a time-delay network. To assess the robustness against perturbations, disturbance was selected as

$$d^T = \begin{bmatrix} 0.5 \sin 40t & 0.6 \sin 35t \end{bmatrix}. \tag{75}$$

The closed-loop system response under disturbance is shown in Figure 3, which result confirms the effectiveness of the controller's performance against disturbance. Therefore, the proposed methodology can be employed to synthesize a controller for stabilization of nonlinear time-delay systems under interval time-varying delays, input saturation, and disturbance.

Example 2
Consider the model of a large-scale chemical reactor [43, 44], consisting of two coupled subsystems, given by

$$\begin{aligned} \dot{z}_{j,1} &= -c_{j,1}^{-1} z_{j,1} - l_{j,1} z_{j,1} + \left((1 - r_{j,2})/v_{i,1}\right) z_{j,1} + h_{j,1}(\tau, z_{3-j,1}, z_{3-j,2}), \\ \dot{z}_{j,2} &= -c_{j,2}^{-1} z_{j,2} - l_{j,2} z_{j,2} + h_{j,2}(\tau, z_{3-j,1}, z_{3-j,2}) + (\phi_{j,2} / v_{j,2}\, sat_j(u_j), \end{aligned} \tag{76}$$

where $z_{j,1}$ and $z_{j,2}$ are concentrations of the first and second reactors for the jth subsystem. Two subsystems are considered, that is, $j = 1, 2$. The constants $c_{j,1}$ and $c_{j,2}$ represent residence times of the reactors, $l_{j,1}$ and $l_{j,2}$ are the reaction rates, $v_{i,1}$ and $v_{i,2}$ denote the reactor volume, $r_{j,2}$ represents the recycle flow rate, and $\phi_{j,2}$ stands for the input pump feed rate. The nonlinear functions $h_{j,1}(\tau, z_{3-j,1}, z_{3-j,2})$ and $h_{j,2}(\tau, z_{3-j,1}, z_{3-j,2})$ are the uncertain delayed functions acting from one subsystem to the other. The large-scale reactor model can be rewritten as (1) by selecting

$$A = \begin{bmatrix} -c_{1,1}^{-1} - l_{1,1} & (1 - r_{1,2})/v_{1,1} & 0 & 0 \\ 0 & -c_{1,2}^{-1} - l_{1,2} & 0 & 0 \\ 0 & 0 & -c_{2,1}^{-1} - l_{2,1} & (1 - r_{2,2})/v_{2,1} \\ 0 & 0 & 0 & -c_{2,2}^{-1} - l_{2,2} \end{bmatrix},$$

$$f(t,x) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \; g(t, x(t-\tau)) = \begin{bmatrix} h_{1,1}(\tau, z_{2,1}, z_{2,2}) \\ h_{1,2}(\tau, z_{2,2}, z_{2,2}) \\ h_{2,1}(\tau, z_{1,1}, z_{1,2}) \\ h_{2,2}(\tau, z_{1,2}, z_{1,2}) \end{bmatrix}, \; B = \begin{bmatrix} 0 & 0 \\ \phi_{1,2}/v_{1,2} & 0 \\ 0 & 0 \\ 0 & \phi_{2,2}/v_{2,2} \end{bmatrix},$$

$A_d = diag(0,0,0,0)$, $C = diag(1,1,1,1)$, $d = \begin{bmatrix} d_1 & d_2 & d_2 & d_4 \end{bmatrix}^T$, and $sat(u) = \begin{bmatrix} sat_1(u_1) & sat_2(u_2) \end{bmatrix}^T$. The model parameters are selected as $c_{1,1} = c_{1,2} = 2, l_{1,1} = l_{1,2} = 0.3, v_{1,1} = v_{1,2} = 0.5, r_{1,2} = \phi_{1,2} = 0.5, c_{2,1} = c_{2,2} = 2.3, l_{2,1} = l_{2,2} = 0.4, v_{2,1} = v_{2,2} = 0.6, r_{2,2} = \phi_{2,2} = 0.6, \bar{u}_1 = \bar{u}_2 = 5, \Lambda_f = 0$, and $\Lambda_g = 0.5$.

To evaluate the performance of the proposed control scheme provided in Theorem 3, various numerical simulations were performed. Table I shows the maximum allowable value of the upper bound on time-delay for fixed values of lower bound on the delay under $\mu = 0.1$. For $\tau_1 = 0$, the maximum value of the upper bound on delay is obtained as $\tau_2 = 3.744 \times 10^2$. It can be deduced from the table that the allowable value of τ_2 increases by increasing the value of lower bound τ_1. Table II depicts the maximum allowable value of delay-rate under $\tau_1 = 10$ by varying the range of delay, that is, τ_{21}. The proposed methodology can allow fast varying time-delays owing to the larger value of tolerable delay-rate μ. It was also observed that the maximum value of μ decreases with the increase in τ_{21}.

In another study, the minimization of parameters ξ and λ is studied to ensure the maximization of the allowable disturbance and minimization of the disturbance effects at the output, respectively, for $\sigma = 200$. The parameters of the time-delay are selected as $\tau_1 = 10, \tau_2 = 20$ and $\mu = 0.5$.

Table I. Maximum values of τ_2 for the given values of lower bound τ_1.

Lower bound τ_1	0	10	20	30	40
Maximum value of τ_2	3.744×10^2	3.748×10^2	3.824×10^2	3.952×10^2	4.489×10^2

Table II. Maximum values of μ for the given values of τ_{21} under $\tau_1 = 10$.

Range of delay τ_{21}	10	20	30	40	50
Maximum value of μ	3.00×10^5	2.91×10^5	2.52×10^5	2.26×10^5	2.10×10^5

Table III. Minimum values of ξ and λ for given value of σ.

Given value of parameter σ	200	200
Minimum value obtained	$\xi = 1.08 \times 10^{-27}$	$\lambda = 0.0044$
Resultant design parameter	$\delta^{-1} = 4.63 \times 10^{24}$	$\gamma = 0.938$

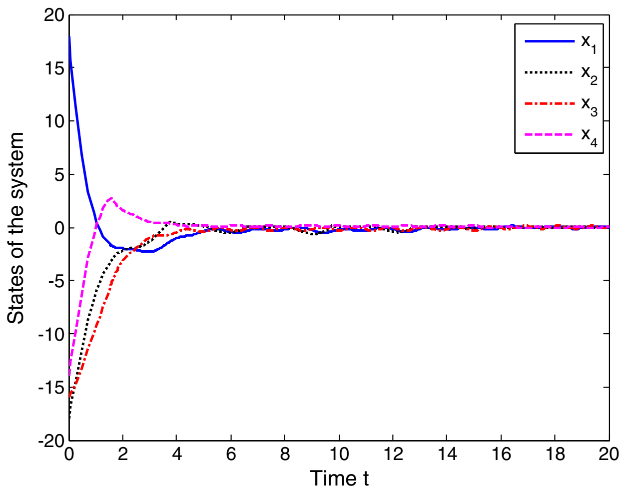

Figure 4. Closed-loop response of the large-scale chemical reactor by means of the proposed method.

Table III shows the minimum values of ξ and λ for the case. It can be concluded that the proposed methodology can offer the minimization of disturbance consequences by minimization of the L_2 gain γ and enhancement of L_2 norm δ^{-1} for disturbance.

In the end, we considered the design of a controller for the large-scale chemical reactor by taking the upper and lower bounds of time-delay as $\tau_1 = 10$ and $\tau_2 = 20$, respectively. The bound on the derivative of the time-varying delay is taken as $\mu = 0.5$. By solving the constraints in Theorem 2 for $\sigma = 200$, $\xi = 6.71 \times 10^{-4}$, and $\lambda = 0.0046$, the controller gain matrix is obtained as

$$F = \begin{bmatrix} -148.75 & -415.18 & 0 & 0 \\ 0 & 0 & -287.86 & -557.21 \end{bmatrix}.$$

The nonlinear perturbations are taken as

$$\begin{aligned} h_{1,1}(\tau, z_{2,1}, z_{2,2}) &= 0.02 z_{2,1}(t-\tau)\cos z_{2,2}(t-\tau), \\ h_{1,2}(\tau, z_{2,2}, z_{2,2}) &= 0.27 z_{2,1}(t-\tau)\sin z_{2,2}(t-\tau), \\ h_{2,1}(\tau, z_{1,1}, z_{1,2}) &= 0.01 z_{1,1}(t-\tau)\tanh z_{1,2}(t-\tau), \\ h_{2,2}(\tau, z_{1,1}, z_{1,2}) &= 0.24 z_{1,1}(t-\tau)\sin z_{1,2}(t-\tau). \end{aligned}$$

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The disturbance and the initial condition are selected as

$$\phi(t) = \begin{bmatrix} 18 + 0.1\sin t \\ -18 - 0.2\sin 1.5t \\ -16 + 0.4\sin 1.3t \\ -14 - 0.3\sin 1.8t \end{bmatrix}, \; d = \begin{bmatrix} 0.5\sin 4t \\ 1.9\sin 2t \\ 0.7\sin 5t \\ 2.2\sin 3t \end{bmatrix}.$$

The closed-loop system response is plotted in Figure 4 for $\tau(t) = 15 + 0.4\cos t$. Under the time-varying delay, disturbances, and input saturation, the states of the large-scale chemical reactor are converging in the neighborhood of the origin. Hence, the proposed methodology can be effectively applied to the control of industrial nonlinear time-delay systems subjected to the input constraint, interval time-varying delays, and external perturbations.

5. CONCLUSIONS

Feedback control of nonlinear time-delay systems under input saturation in the presence of interval time-varying delays was investigated. A sufficient condition ensuring local stability of the input-constrained nonlinear time-delay systems and providing an estimate of the region of convergence was formulated by employing the local sector condition, a Lyapunov–Krasovskii functional, and nonlinearity bounds. The proposed controller design treatment's novelty and effectiveness, even for application to linear time-delay systems, was demonstrated. Further, a new delay-dependent controller formation approach to nonlinear time-delay systems subject to input saturation was inferred as a particular case of the proposed control methodology. Moreover, the resultant delay-range-dependent approach for stabilization of input-saturated nonlinear time-delay systems with time-varying delays was broadened for disturbance rejection by ensuring L_2 gain reduction between the disturbance and output vectors. The proposed stabilization methodology represents a modest effort to fill the research gap in delay-range-dependent control of nonlinear systems under input saturation; it will certainly reveal, in any case, further fruitful research avenues for exploration. In future, delay-range-dependent data-driven approaches for delayed nonlinear systems can be developed to deal with the actuator saturation faults. Simulation results indicated a satisfactory performance for stabilization of an unstable input-constrained nonlinear time-delay system and control of a large-scale chemical reactor with time-delays varying between lower and upper bounds.

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